Pool Ball Arrangement

Tara Fahey

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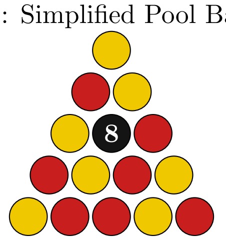
1 Introduction

Figure 1: Simplified Pool Ball Rack



I had a question posed to me: while investigating a random sequence of red yellow balls, where there are even number of red and yellow balls, what's 1) The maximum number of swaps needed to transform it into an alternating sequence, and 2) What's the expected number of swaps?

Our sequence may look like so:

RRYRRYYY

Whereas we want to convert it to either

RYRYRYRY

YRYRYRYR

An algorithm I propose for this is as follows. Split the sequence into sequential groups of two. They will be either RR, YY, RY, or YR.

To achieve the minimum number of swaps, our alternating sequence will be made up of whichever of RY or YR is more common. Assume w.l.o.g. that RY is more common.

Then any group of YR can become RY with one swap. And for every RR, there will be another YY, so we can swap the second R with the first Y to create two new RY pairs.

Lemma 1. *There are equal numbers of RR as YY*

	YY	RR	RY	YR
Swaps	$\frac{1}{2}$	$\frac{1}{2}$	0	1
	RY	RY	RY	RY

Proof. Take any sequence of red and yellow balls with equal amounts of red and yellow. We remove all the instances of RY YR, and will still have equal amounts of red and yellow balls left, as we've removed red and yellow at the same time. Thus we only have RR and YY left, where number of red balls equals number of yellow balls, and thus the number of RR and YY pairs are also equal. □

Claim 1. *The max number of swaps necessary is $\frac{n}{2}$.*

Proof. Split sequence into one group of RR & YY's of length l_1, and RY & YR of length l_2 Then RR & YY group will need $\frac{l_1}{2}$ swaps And RY & YR will need $S_2 = min(|RY|, |YR|)$ swaps. Since $|RY| + |YR| = l_2$, $S_2 < \frac{l_2}{2}$

$Swaps < \frac{l_1}{2} + \frac{l_2}{2}$

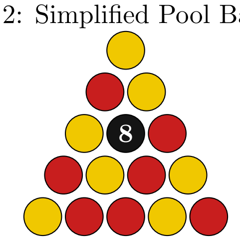
$\therefore Swaps < \frac{n}{2}$

□

2 Extension to Pool Balls

We return to the original question: what's the minimum number of swaps needed to put a pool table into its correct configuration?

Figure 2: Simplified Pool Ball Rack



If we find an alternating pattern in the required configuration, we can apply our above theorem.

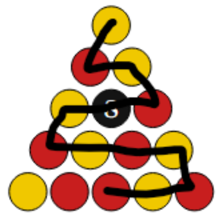


Figure 3: Route

We'll require one swap to put the black ball in the center, and one swap to achieve the bottom left configuration that doesn't fit our alternating sequence.